

Ecole Polytechnique
ECO 560 Economics of Contracts

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Lecture 6: Moral Hazard

1 Introduction

This lecture we consider the models with hidden action, also known as models with moral hazard. In this models, the Principal does not observe the actions taken by the Agent. From the perspective of a simple Stackelberg game, the uninformed Principal acts first by offering a contract and then the Agent takes an action (usually, chooses effort) that is observed only by her. The Principal can condition the contract only on noisy output signals about the Agent's action. In this sense, the problem, of the Principal looks remarkably looks like a problem about statistical inference. We will investigate several variations of the basic model depending on the criterion functions used by the Principal and the Agent. We will find that in most cases, the solution of the moral hazard problem is very simple. Complications arise only when the Agent are risk-averse. When this happens, there is a trade-off between inducing the Agent to work and offering some form of insurance against the variation in the output signals. Unfortunately, the general formulation of the model leads to very few testable predictions.

2 A Simple Model

Suppose that the output signal y can take only two values, 0 and 1. Zero means failure and 1 a success. The Agent takes an action e that affects the

probability of success and failure. Let

$$\Pr(y = 1|e) = \pi(e)$$

be strictly increasing and concave in e , such that

$$\pi(0) = 0, \pi(\infty) = 1, \pi'(0) > 1$$

The criterion function of the principal is

$$V(y - w)$$

where w is the compensation offered to the Agent. The criterion function of the Principal obeys the following restrictions:

$$V' > 0, V'' \leq 0$$

That is, it is increasing in the output signal and the Principal can be potentially risk-averse. The criterion function of the Agent is

$$u(w) - \psi(e)$$

where

$$u' > 0, u'' \leq 0, \psi' > 0, \psi'' \geq 0$$

Note that this specification implies that the criterion function of the Agent is separable in income and the action. Separability eliminates any income effects that may influence the choice of action through the marginal cost of the action. In the general model that we will consider later, it guarantees that the Agent's individual rationality constraint is always binding at the optimum.

3 Observable and Verifiable Actions

When the actions of the Agent are observable and verifiable by a third party, the Agent's compensation can be conditioned on the chosen action. In what follows we will focus on finding the optimal contract from the perspective of the Principal. The Principal's problem is then to choose an action and compensations, (e, w_1, w_0) that solves:

$$\max_{(e, w_i)} \pi(e) V(1 - w_1) + (1 - \pi(e)) V(-w_0)$$

subject to

$$\pi(e)u(w_1) + (1 - \pi(e))u(w_0) - \psi(e) = \underline{u}$$

where \underline{u} indicates the value of the Agent's criterion function for an outside option. Without loss of generality, we maintain that it is 0. Let λ be the Lagrange multiplier associated with the constraint. The first order conditions with respect to w_1 and w_0 are

$$\pi(e) \cdot (-V'(1 - w_1) + \lambda u'(w_1)) = 0$$

$$(1 - \pi(e)) \cdot (-V'(-w_0) + \lambda u'(w_0)) = 0$$

These two conditions imply that

$$\frac{V'(1 - w_1)}{u'(w_1)} = \frac{V'(-w_0)}{u'(w_0)} = \lambda$$

Note that this condition states that the ratio of the marginal subjective value of the rewards to the Principal and Agent must be equal at the optimum across states of the world. Moreover, the Lagrange multiplier defines this subjective valuation. This condition is also known as the Borch rule.

The first order condition with respect to e is:

$$\pi'(e) [(V(1 - w_1) - V(-w_0)) + \lambda(u(w_1) - u(w_0))] = \lambda\psi'(e)$$

This condition, along with the individual rationality constraint, and the Borch rule, can be used to solve for the optimal action a . Next, we consider several cases of the problem.

3.1 Risk-Neutral Principal

Suppose that the Principal is risk-neutral and $V(x) = x$. Then the first order conditions from above and the individual rationality constraint imply that the Agent is paid the same wage across states, w^* and the optimal bundle (w^*, e^*) is the solution to the following system of equations:

$$\begin{aligned} u(w) &= \underline{u} \\ \pi'(e) &= \frac{\psi'(e)}{u'(w)} \end{aligned}$$

That is, at the optimum the marginal productivity of effort is equal to its cost. The key feature here is that the employment contract offers full insurance to the Agent against the fluctuations in output.

3.2 Risk-Neutral Agent

Now suppose that the Agent is risk-neutral with $u(x) = x$. The optimality conditions imply that the Principal is fully ensured with

$$w_1 - w_0 = 1$$

$$\pi'(e) = \frac{\psi'(e)}{(u(w_1) - u(w_0))}$$

Thus, when the Agent is risk-neutral, the optimal contract specifies that the Principal is fully insured against the fluctuations in output.

4 Unobservable or Unverifiable Actions

When actions are not observable or verifiable, the optimal contract cannot condition on the action. The only way for the Principal to induce the Agent to exert effort is by conditioning pay on the noisy outcome. This introduces a tension between providing incentives to exert effort and providing insurance. The Agent chooses an action to maximize her expected utility:

$$\max_e \pi(e) u(w_1) + (1 - \pi(e)) u(w_0) - \psi(e)$$

The problem of the Principal is then:

$$\max_{(e, w_i)} \pi(e) V(1 - w_1) + (1 - \pi(e)) V(-w_0)$$

subject to

$$\pi(e) u(w_1) + (1 - \pi(e)) u(w_0) - \psi(e) = \underline{u}$$

and

$$e \in \arg \max_{(e, w_i)} \pi(e) V(1 - w_1) + (1 - \pi(e)) V(-w_0)$$

Let us assume for the moment that the second-order conditions of the Agent's problem are satisfied. Then the first-order condition for the Agent's program provide the necessary and sufficient conditions to find the optimal action e , given (w_1, w_0) . Thus, we can replace the last constraint in the

problem of the Principal with the first-order condition for the problem of the Agent. Thus, the problem of the Principal becomes:

$$\max_{(e, w_i)} \pi(e) V(1 - w_1) + (1 - \pi(e)) V(-w_0)$$

subject to

$$\begin{aligned} \pi(e) u(w_1) + (1 - \pi(e)) u(w_0) - \psi(e) &= \underline{u} \\ \pi'(e) (u(w_1) - u(w_0)) &= \psi'(e) \end{aligned}$$

To simplify the presentation, assume that $\psi(e) = e$.

4.1 Risk-Neutral Agent and Principal

As before, suppose that the Agent's utility takes the form $u(x) = x$ and for the Principal $V(x) = x$. Then, the first order conditions under perfect information implied that:

$$\pi'(e) = 1$$

Here we also have that

$$\pi'(e) (w_1 - w_0) = 1$$

which implies that when

$$w_1 - w_0 = 1$$

the principal can implement the first best action. One way to interpret this statement is that the Principal 'sells' up-front to the Agent the ownership rights over the output. In this way, the Agent owns the output and chooses an action to maximize her utility without any constraints. The price at which the Principal sells the rights to the output is the first-best $-w_0$.

Now consider a case in which there is a restriction on w_0 , say $w_0 \geq 0$. When $w_0 = 0$, the Agent's utility is:

$$u(e) = \pi(e) - e$$

At the optimum, $u(e)$ is positive because $\pi''(e) < 0$ and

$$\pi'(e) = 1$$

so the Agent will accept $w_0 = 0$. The Principal clearly would like to sell the rights at price $w_0 = 0$.

Again, from the first order conditions for the Principal, it follows that

$$\pi'(e) w_1 = 1$$

What remains is to solve for the optimal effort choice e . The problem of the Principal becomes then:

$$\max_e \pi(e) (1 - w_1)$$

subject to

$$\pi'(e) w_1 = 1$$

The first order conditions for this problem imply that the optimal e is the solution of

$$\pi'(e) = 1 - \frac{\pi'(e) \pi''(e)}{[\pi'(e)]^2}$$

Since, we have assumed that π is strictly increasing and strictly concave, this result implies that the right hand side of the equation is greater than 1, which implies that the optimal effort e is less than the optimal effort in the case of observable and verifiable actions.

4.1.1 Interpretation

One environment in which this model can be applied is the management of large companies. There, the manager of the firm is the Agent and the Principal is the investors in the firm. In this case, $(1 - w_1)$ can be thought as the outstanding debt of the company which has been financed by the investors, and w_1 is the reward to the managers. In such a case, lowering w_1 and increasing the debt burden leads to lower incentives to exert effort which implies that Principal is facing some form of Laffer curve: reducing the debt burden can lead to an increase in the debts real present value.

4.2 Risk Averse Agent

Consider the problem when the Agent is risk averse and actions are unobservable and unverifiable. The problem of the Principal is then:

$$\max_{(e, w_i)} \pi(e) V(1 - w_1) + (1 - \pi(e)) V(-w_0)$$

subject to

$$\pi(e) u(w_1) + (1 - \pi(e)) u(w_0) - \psi(e) = \underline{u} \quad (\lambda)$$

$$\pi'(e) (u(w_1) - u(w_0)) = 1 \quad (\mu)$$

The first order conditions are then

$$\pi(e) \cdot (-V'(1 - w_1) + \lambda u'(w_1)) + \mu \pi'(e) u'(w_1) = 0$$

$$(1 - \pi(e)) \cdot (-V'(-w_0) + \lambda u'(w_0)) - \mu \pi'(e) u'(w_0) = 0$$

$$\pi'(e) [(V(1 - w_1) - V(-w_0)) + \lambda (u(w_1) - u(w_0))] + \pi''(e) (u(w_1) - u(w_0)) = \lambda$$

The first two constraints can be rewritten as

$$\frac{V'(1 - w_1)}{u'(w_1)} = \lambda + \mu \frac{\pi'(e)}{\pi(e)}$$

$$\frac{V'(-w_0)}{u'(w_0)} = \lambda - \mu \frac{\pi'(e)}{1 - \pi(e)}$$

This reduces to the original Borch rule when $\mu = 0$. However, $\mu > 0$ under general conditions that we will investigate in the following lecture. These first order conditions allow us to characterize some features of the optimal contract that we will discuss in greater detail later.

4.3 Comments on the solution

- $u(w_1) - u(w_0) > 0$ so the worker receives more when a good outcome is observed.
- Since $\pi(e)$ is strictly increasing and concave, e under observed and verifiable actions is higher than the optimal effort under moral hazard. This property, however, depends heavily on the distributional assumptions.
- The principal predicts perfectly what the chosen action will be.
- Thus variation in observed compensation is driven entirely by luck.

- Nevertheless, the optimal contract specifies different compensation based on outcome in order to induce exerting effort.

This simple framework presents some of the important main results, but at the same time obscures some issues:

- The problem of the Principal is closely related to statistical inference based on the observed signals. We imposed some very strong assumptions on the conditional distribution of success given effort. What minimal conditions will be necessary for the solution of the problem to retain the main features outlined above?
- When does using the first order condition of the Agent's problem is legitimate?